CHAPTER 17 (Odd)

3. a. $\mathbf{Z} = 15\ \Omega - j16\ \Omega = \mathbf{21.93\ \Omega \angle -46.85^\circ}$
$\mathbf{E} = \mathbf{IZ} = (0.5\ \text{A} \angle 60^\circ)(21.93\ \Omega \angle -46.85^\circ)$
$= \mathbf{10.97\ V \angle 13.15^\circ}$

b. $\mathbf{Z} = 10\ \Omega \angle 0^\circ \| 6\ \Omega \angle 90^\circ = \mathbf{5.15\ \Omega \angle 59.04^\circ}$
$\mathbf{E} = \mathbf{IZ} = (2\ \text{A} \angle 120^\circ)(5.15\ \Omega \angle 59.04^\circ)$
$= \mathbf{10.30\ V \angle 179.04^\circ}$

5. a. Clockwise mesh currents:

$$\mathbf{E} - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 = 0$$
$$-\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

$\mathbf{Z}_1 = R_1 \angle 0^\circ = 4\ \Omega \angle 0^\circ$
$\mathbf{Z}_2 = X_L \angle 90^\circ = 6\ \Omega \angle 90^\circ$
$\mathbf{Z}_3 = X_C \angle -90^\circ = 8\ \Omega \angle -90^\circ$
$\mathbf{E}_1 = 10\ \text{V} \angle 0^\circ,\ \mathbf{E}_2 = 40\ \text{V} \angle 60^\circ$

$$[\mathbf{Z}_1 + \mathbf{Z}_2]\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1$$
$$-\mathbf{Z}_2\mathbf{I}_1 + [\mathbf{Z}_2 + \mathbf{Z}_3]\mathbf{I}_2 = -\mathbf{E}_2$$

$$\mathbf{I}_{R_1} = \mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & [\mathbf{Z}_2 + \mathbf{Z}_3] \end{vmatrix}}{\begin{vmatrix} [\mathbf{Z}_1 + \mathbf{Z}_2] & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & [\mathbf{Z}_2 + \mathbf{Z}_3] \end{vmatrix}} = \frac{[\mathbf{Z}_2 + \mathbf{Z}_3]\mathbf{E}_1 - \mathbf{Z}_2\mathbf{E}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} = \mathbf{5.15\ A \angle -24.5^\circ}$$

b. By interchanging the right two branches, the general configuration of part (a) will result and

$$\mathbf{I}_{50\Omega} = \mathbf{I}_1 = \frac{[\mathbf{Z}_2 + \mathbf{Z}_3]\mathbf{E}_1 - \mathbf{Z}_2\mathbf{E}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$
$$= \mathbf{0.442\ A \angle 143.48^\circ}$$

$\mathbf{Z}_1 = R_1 = 50\ \Omega \angle 0^\circ$
$\mathbf{Z}_2 = X_C \angle -90^\circ = 60\ \Omega \angle -90^\circ$
$\mathbf{Z}_3 = X_L \angle 90^\circ = 20\ \Omega \angle 90^\circ$
$\mathbf{E}_1 = 5\ \text{V} \angle 30^\circ,\ \mathbf{E}_2 = 20\ \text{V} \angle 0^\circ$

7. a. Clockwise mesh currents:

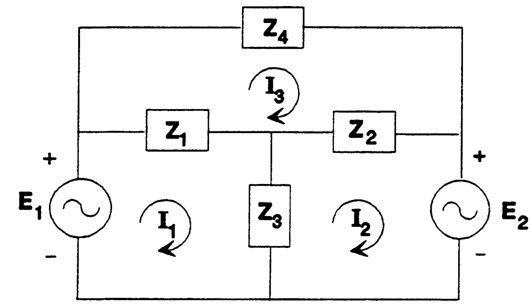
$$\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 = 0$$
$$-\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_2\mathbf{Z}_4 + \mathbf{I}_3\mathbf{Z}_4 = 0$$
$$-\mathbf{I}_3\mathbf{Z}_4 + \mathbf{I}_2\mathbf{Z}_4 - \mathbf{I}_3\mathbf{Z}_5 - \mathbf{E}_2 = 0$$

$\mathbf{Z}_1 = 4\ \Omega + j3\ \Omega,\ \mathbf{Z}_2 = -j1\ \Omega$
$\mathbf{Z}_3 = +j6\ \Omega,\ \mathbf{Z}_4 = -j2\ \Omega$
$\mathbf{Z}_5 = 8\ \Omega$
$\mathbf{E}_1 = 60\ \text{V} \angle 0^\circ,\ \mathbf{E}_2 = 120\ \text{V} \angle 120^\circ$

$$\begin{aligned}
&[\mathbf{Z}_1 + \mathbf{Z}_2]\mathbf{I}_1 && - \mathbf{Z}_2\mathbf{I}_2 && + 0 = \mathbf{E}_1 \\
&-\mathbf{Z}_2\mathbf{I}_1 && + [\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4]\mathbf{I}_2 && - \mathbf{Z}_4\mathbf{I}_3 = 0 \\
&0 && - \mathbf{Z}_4\mathbf{I}_2 && + [\mathbf{Z}_4 + \mathbf{Z}_5]\mathbf{I}_3 = -\mathbf{E}_2
\end{aligned}$$

$$\mathbf{I}_{R_1} = \mathbf{I}_3 = \frac{[\mathbf{Z}_2\mathbf{Z}_4]\mathbf{E}_1 + \left[\mathbf{Z}_2^2 - [\mathbf{Z}_1 + \mathbf{Z}_2][\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4]\right]\mathbf{E}_2}{[\mathbf{Z}_1 + \mathbf{Z}_2][\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4][\mathbf{Z}_4 + \mathbf{Z}_5] - [\mathbf{Z}_1 + \mathbf{Z}_2]\mathbf{Z}_4^2 - [\mathbf{Z}_4 + \mathbf{Z}_5]\mathbf{Z}_2^2}$$
$$= \mathbf{13.07\ A \angle -33.71^\circ}$$

b.



$\mathbf{Z}_1 = 15\ \Omega\ \angle 0°,\ \mathbf{Z}_2 = 15\ \Omega\ \angle 0°$
$\mathbf{Z}_3 = -j10\ \Omega = 10\ \Omega\ \angle -90°$
$\mathbf{Z}_4 = 3\ \Omega + j4\ \Omega = 5\ \Omega\ \angle 53.13°$
$\mathbf{E}_1 = 220\ \text{V}\ \angle 0°$
$\mathbf{E}_2 = 100\ \text{V}\ \angle 90°$

$$\begin{aligned}
&\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_3) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_3\mathbf{Z}_1 = \mathbf{E}_1\\
&\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_3 - \mathbf{I}_3\mathbf{Z}_2 = -\mathbf{E}_2\\
&\mathbf{I}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 = 0
\end{aligned}$$

$$\begin{array}{llll}
\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_3) & - \mathbf{I}_2\mathbf{Z}_3 & - \mathbf{I}_3\mathbf{Z}_1 & = \mathbf{E}_1\\
-\mathbf{I}_1\mathbf{Z}_3 & + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) & - \mathbf{I}_3\mathbf{Z}_2 & = -\mathbf{E}_2\\
-\mathbf{I}_1\mathbf{Z}_1 & - \mathbf{I}_2\mathbf{Z}_2 & + \mathbf{I}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4) & = 0
\end{array}$$

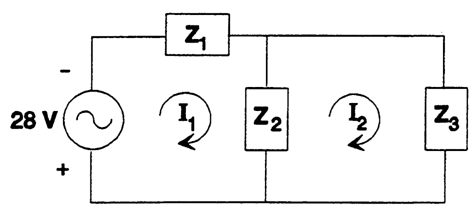
Applying determinants:

$$\mathbf{I}_3 = \frac{-(\mathbf{Z}_1+\mathbf{Z}_3)(\mathbf{Z}_2)\mathbf{E}_2 - \mathbf{Z}_1\mathbf{Z}_3\mathbf{E}_2 + \mathbf{E}_1[\mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1(\mathbf{Z}_2+\mathbf{Z}_3)]}{(\mathbf{Z}_1+\mathbf{Z}_3)\left[(\mathbf{Z}_2+\mathbf{Z}_3)(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_4) - \mathbf{Z}_2^2\right] + \mathbf{Z}_3[\mathbf{Z}_3(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_4) - \mathbf{Z}_1\mathbf{Z}_2] - \mathbf{Z}_1[-\mathbf{Z}_2\mathbf{Z}_3 - \mathbf{Z}_1(\mathbf{Z}_2+\mathbf{Z}_3)]}$$

$= \mathbf{48.33\ A\ \angle -77.57°}$

or $\mathbf{I}_3 = \dfrac{\mathbf{E}_1 - \mathbf{E}_2}{\mathbf{Z}_4}$ if one carefully examines the network!

9.



$\mathbf{Z}_1 = 5\ \text{k}\Omega\ \angle 0°$
$\mathbf{Z}_2 = 10\ \text{k}\Omega\ \angle 0°$
$\mathbf{Z}_3 = 1\ \text{k}\Omega + j4\ \text{k}\Omega = 4.123\ \text{k}\Omega\ \angle 75.96°$

$$\begin{aligned}
&\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{Z}_2\mathbf{I}_2 = -28\ \text{V}\\
&\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_2\mathbf{I}_1 = 0
\end{aligned}$$

$$\begin{aligned}
&(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = -28\ \text{V}\\
&-\mathbf{Z}_2\mathbf{I}_1 + (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_2 = 0
\end{aligned}$$

$$\mathbf{I}_L = \mathbf{I}_2 = \frac{-\mathbf{Z}_2\ 28\ \text{V}}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} = \mathbf{-3.165 \times 10^{-3}\ V\ \angle 137.29°}$$

11. $6\mathbf{V}_x - \mathbf{I}_1\ 1\ \text{k}\Omega - 10\ \text{V}\ \angle 0° = 0$
$10\ \text{V}\angle 0° - \mathbf{I}_2\ 4\ \text{k}\Omega - \mathbf{I}_2\ 2\ \text{k}\Omega = 0$

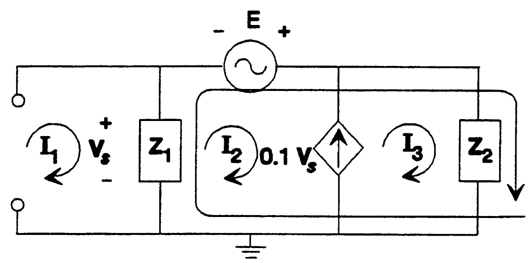
$\mathbf{V}_x = \mathbf{I}_2\ 2\ \text{k}\Omega$

$-\mathbf{I}_1\ 1\ \text{k}\Omega + \mathbf{I}_2\ 12\ \text{k}\Omega = 10\ \text{V}\ \angle 0°$
$-\mathbf{I}_2\ 6\ \text{k}\Omega = -10\ \text{V}\ \angle 0°$

$$I_2 = \frac{10\text{ V}\angle 0°}{6\text{ k}\Omega} = 1.667\text{ mA}\angle 0° = I_{2k\Omega}$$

$$-I_1\,1\text{ k}\Omega + (1.667\text{ mA}\angle 0°)(12\text{ k}\Omega) = 10\text{ V}\angle 0°$$
$$-I_1\,1\text{ k}\Omega + 20\text{ V}\angle 0° = 10\text{ V}\angle 0°$$
$$-I_1\,1\text{ k}\Omega = -10\text{ V}\angle 0°$$
$$I_1 = \frac{10\text{ V}\angle 0°}{1\text{ k}\Omega} = 10\text{ mA}\angle 0°$$

13.



$Z_1 = 1\text{ k}\Omega\angle 0°$
$Z_2 = 4\text{ k}\Omega + j6\text{ k}\Omega$
$E = 10\text{ V}\angle 0°$

$$-Z_1(I_2 - I_1) + E - I_3Z_3 = 0$$
$$I_1 = 6\text{ mA}\angle 0°,\ 0.1\,V_s = I_3 - I_2,\ V_s = (I_1 - I_2)Z_1$$

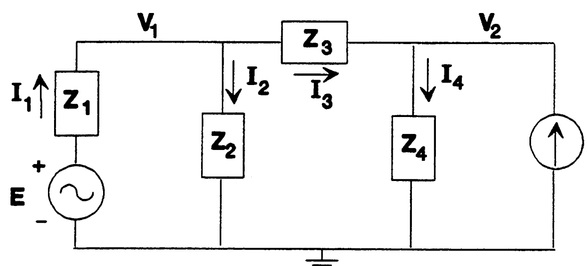
Substituting:

$$(1\text{ k}\Omega)I_2 + (4\text{ k}\Omega + j6\text{ k}\Omega)I_3 = 16\text{ V}\angle 0°$$
$$(99\ \Omega)I_2 + I_3 = 0.6\text{ V}\angle 0°$$

Determinants:

$$I_3 = I_{6\text{ k}\Omega(2)} = 1.378\text{ mA}\angle -56.31°$$

15. a.



$Z_1 = 5\ \Omega\angle 0°$
$Z_2 = 6\ \Omega\angle 90°$
$Z_3 = 4\ \Omega\angle -90°$
$Z_4 = 2\ \Omega\angle 0°$
$E = 30\text{ V}\angle 50°$
$I = 0.04\text{ A}\angle 90°$

$$I_1 = I_2 + I_3$$
$$\frac{E_1 - V_1}{Z_1} = \frac{V_1}{Z_2} + \frac{(V_1 - V_2)}{Z_3} \Rightarrow V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - \frac{V_2}{Z_3} = \frac{E_1}{Z_1}$$
or $V_1[Y_1 + Y_2 + Y_3] - Y_3V_2 = E_1Y_1$

$$I_3 + I = I_4$$
$$\frac{V_1 - V_2}{Z_3} + I = \frac{V_2}{Z_4} \Rightarrow V_2\left[\frac{1}{Z_3} + \frac{1}{Z_4}\right] - \frac{V_1}{Z_3} = I$$

or $V_2[Y_3 + Y_4] - V_1Y_3 = I$

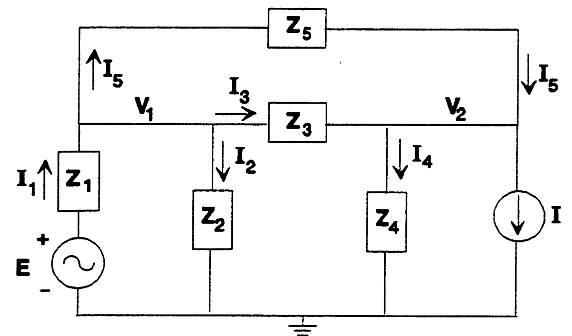
resulting in

$$\begin{aligned} \mathbf{V}_1[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3] - \mathbf{V}_2\mathbf{Y}_3 &= \mathbf{E}_1\mathbf{Y}_1 \\ -\mathbf{V}_1[\mathbf{Y}_3] + \mathbf{V}_2[\mathbf{Y}_3 + \mathbf{Y}_4] &= +\mathbf{I} \end{aligned}$$

Using determinants:

$$\mathbf{V}_1 = \mathbf{19.86\ V\ \angle 43.8^\circ} \text{ and } \mathbf{V}_2 = \mathbf{8.94\ V\ \angle 106.9^\circ}$$

b.



$\mathbf{Z}_1 = 10\ \Omega\ \angle 0^\circ$
$\mathbf{Z}_2 = 10\ \Omega\ \angle 0^\circ$
$\mathbf{Z}_3 = 4\ \Omega\ \angle 90^\circ$
$\mathbf{Z}_4 = 2\ \Omega\ \angle 0^\circ$
$\mathbf{Z}_5 = 8\ \Omega\ \angle -90^\circ$
$\mathbf{E} = 50\ \text{V}\ \angle 120^\circ$
$\mathbf{I} = 0.8\ \text{A}\ \angle 70^\circ$

$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_5$

$$\frac{\mathbf{E} - \mathbf{V}_1}{\mathbf{Z}_1} = \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{\mathbf{Z}_5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} \Rightarrow \mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_5}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_5}\right] = \frac{\mathbf{E}}{\mathbf{Z}_1}$$

or $\mathbf{V}_1[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_5] - \mathbf{V}_2[\mathbf{Y}_3 + \mathbf{Y}_5] = \mathbf{E}_1\mathbf{Y}_1$

$\mathbf{I}_3 + \mathbf{I}_5 = \mathbf{I}_4 + \mathbf{I}$

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_5} = \frac{\mathbf{V}_2}{\mathbf{Z}_4} + \mathbf{I} \Rightarrow \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4} + \frac{1}{\mathbf{Z}_5}\right] - \mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_5}\right] = -\mathbf{I}$$

or $\mathbf{V}_2[\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_5] - \mathbf{V}_1[\mathbf{Y}_3 + \mathbf{Y}_5] = -\mathbf{I}$

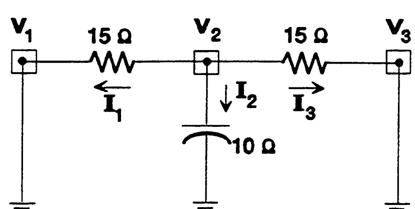
resulting in

$$\begin{aligned} \mathbf{V}_1[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_5] - \mathbf{V}_2[\mathbf{Y}_3 + \mathbf{Y}_5] &= \mathbf{E}_1\mathbf{Y}_1 \\ -\mathbf{V}_1[\mathbf{Y}_3 + \mathbf{Y}_5] + \mathbf{V}_2[\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_5] &= -\mathbf{I} \end{aligned}$$

Applying determinants:

$$\mathbf{V}_1 = \mathbf{19.78\ V\ \angle 132.48^\circ} \text{ and } \mathbf{V}_2 = \mathbf{13.37\ V\ \angle 98.78^\circ}$$

17.



(Note that 3 + j4 branch has no effect on nodal voltages)

$\sum \mathbf{I}_i = \sum \mathbf{I}_o$

$$0 = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = \frac{\mathbf{V}_2 - \mathbf{V}_1}{15\ \Omega} + \frac{\mathbf{V}_2}{10\ \Omega\ \angle -90^\circ} + \frac{\mathbf{V}_2 - \mathbf{V}_3}{15\ \Omega}$$

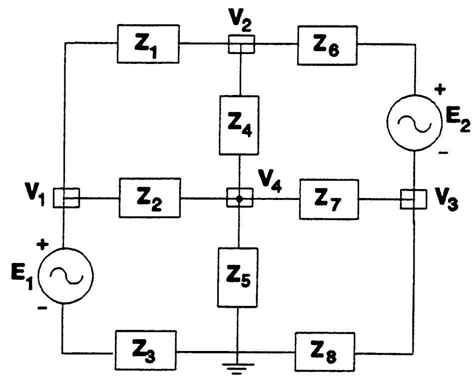
Through manipulation:

$$\mathbf{V}_2[2 + j1.5] - \mathbf{V}_1 - \mathbf{V}_3 = 0$$

but $\mathbf{V}_1 = \mathbf{220\ V\ \angle 0^\circ}$ and $\mathbf{V}_3 = \mathbf{100\ V\ \angle 90^\circ}$

and $\mathbf{V}_2 = \dfrac{220 + j100}{2 + j1.5} = \mathbf{96.664\ V\ \angle -12.426^\circ}$

19.



$E_1 = 25\text{ V}\angle 0°$
$E_2 = 75\text{ V}\angle 20°$

$Z_1 = 10\ \Omega + j20\ \Omega$
$Z_2 = 6\ \Omega\ \angle 0°$
$Z_3 = 5\ \Omega\ \angle 0°$
$Z_4 = 20\ \Omega\ \angle -90°$
$Z_5 = 10\ \Omega\ \angle 0°$
$Z_6 = 80\ \Omega\ \angle 0°$
$Z_7 = 15\ \Omega\ \angle 90°$
$Z_8 = 5\ \Omega - j20\ \Omega$

$$V_1: \frac{V_1 - V_2}{Z_1} + \frac{V_1 - V_4}{Z_2} + \frac{V_1 - E_1}{Z_3} = 0$$

$$V_2: \frac{V_2 - V_1}{Z_1} + \frac{V_2 - V_4}{Z_4} + \frac{V_2 - E_2 - V_3}{Z_6} = 0$$

$$V_3: \frac{V_3 + E_2 - V_2}{Z_6} + \frac{V_3 - V_4}{Z_7} + \frac{V_3}{Z_8} = 0$$

$$V_4: \frac{V_4 - V_1}{Z_2} + \frac{V_4 - V_2}{Z_4} + \frac{V_4 - V_3}{Z_7} + \frac{V_4}{Z_5} = 0$$

Rearranging:

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - \frac{V_2}{Z_1} - \frac{V_4}{Z_2} = \frac{E_1}{Z_3}$$

$$V_2\left[\frac{1}{Z_1} + \frac{1}{Z_4} + \frac{1}{Z_6}\right] - \frac{V_1}{Z_1} - \frac{V_4}{Z_4} - \frac{Z_3}{Z_6} = \frac{E_2}{Z_6}$$

$$V_3\left[\frac{1}{Z_6} + \frac{1}{Z_7} + \frac{1}{Z_8}\right] - \frac{V_2}{Z_6} - \frac{V_4}{Z_7} = -\frac{E_2}{Z_6}$$

$$V_4\left[\frac{1}{Z_2} + \frac{1}{Z_4} + \frac{1}{Z_7} + \frac{1}{Z_5}\right] - \frac{V_1}{Z_2} - \frac{V_2}{Z_4} - \frac{Z_3}{Z_7} = 0$$

Setting up and then using determinants:

$$\mathbf{V_1 = 14.62\text{ V}\ \angle -5.861°,\ V_2 = 35.03\text{ V}\ \angle -37.69°}$$
$$\mathbf{V_3 = 32.4\text{ V}\ \angle -73.34°,\ V_4 = 5.667\text{ V}\ \angle 23.53°}$$

21. Left node: V_1

$$\Sigma I_i = \Sigma I_o$$

$$4I_x = I_x + 5\text{ mA}\ \angle 0° + \frac{V_1 - V_2}{2\text{ k}\Omega}$$

Right node: V_2

$$\Sigma I_i = \Sigma I_o$$

$$8\text{ mA}\ \angle 0° = \frac{V_2}{1\text{ k}\Omega} + \frac{V_2 - V_1}{2\text{ k}\Omega} + 4I_x$$

Insert $I_x = \dfrac{V_1}{4\text{ k}\Omega\ \angle -90°}$

Rearrange, reduce and 2 equations with 2 unknowns result:

$$\mathbf{V}_1[1.803 \angle 123.69°] + \mathbf{V}_2 = 10$$
$$\mathbf{V}_1[2.236 \angle 116.57°] + 3\,\mathbf{V}_2 = 16$$

Determinants: $\mathbf{V}_1 = \mathbf{4.372\ V \angle -128.655°}$
$\mathbf{V}_2 = \mathbf{2.253\ V \angle 17.628°}$

23. Left node: $\mathbf{V}_1$

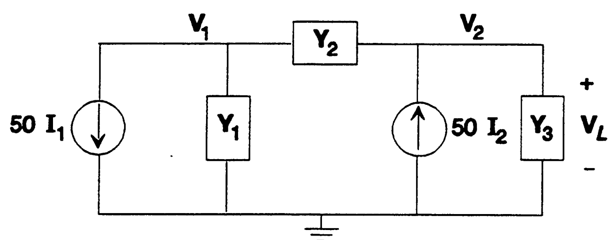
$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$
$$2\text{ mA} \angle 0° = 12\text{ mA} \angle 0° + \frac{\mathbf{V}_1}{2\text{ k}\Omega} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1\text{ k}\Omega}$$
and $1.5\,\mathbf{V}_1 - \mathbf{V}_2 = -10$

Right node: $\mathbf{V}_2$

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$
$$0 = 2\text{ mA} \angle 0° + \frac{\mathbf{V}_2 - \mathbf{V}_1}{1\text{ k}\Omega} - \frac{\mathbf{V}_2 - 6\,\mathbf{V}_x}{3.3\text{ k}\Omega}$$
and $2.7\,\mathbf{V}_1 - 3.7\,\mathbf{V}_2 = -6.6$

Using determinants: $\mathbf{V}_1 = \mathbf{-10.667\ V \angle 0° = 10.667\ V \angle 180°}$
$\mathbf{V}_2 = \mathbf{-6\ V \angle 0° = 6\ V \angle 180°}$

25.



$$\mathbf{I}_1 = \frac{E_i \angle \theta}{R_1 \angle 0°} = 1 \times 10^{-3}\,\mathbf{E}_i$$
$$\mathbf{Y}_1 = \frac{1}{50\text{ k}\Omega} = 0.02\text{ mS} \angle 0°$$
$$\mathbf{Y}_2 = \frac{1}{1\text{ k}\Omega} = 1\text{ mS} \angle 0°$$
$$\mathbf{Y}_3 = 0.02\text{ mS} \angle 0°$$
$$\mathbf{I}_2 = (\mathbf{V}_1 - \mathbf{V}_2)\mathbf{Y}_2$$

$$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{Y}_2\mathbf{V}_2 = -50\mathbf{I}_1$$
$$\mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{Y}_2\mathbf{V}_1 = 50\mathbf{I}_2 = 50(\mathbf{V}_1 - \mathbf{V}_2)\mathbf{Y}_2 = 50\mathbf{Y}_2\mathbf{V}_1 - 50\mathbf{Y}_2\mathbf{V}_2$$

$$(\mathbf{Y}_1 + \mathbf{Y}_2)\mathbf{V}_1 - \mathbf{Y}_2\mathbf{V}_2 = -50\mathbf{I}_1$$
$$-51\mathbf{Y}_2\mathbf{V}_1 + (51\mathbf{Y}_2 + \mathbf{Y}_3)\mathbf{V}_2 = 0$$

$$\mathbf{V}_L = \mathbf{V}_2 = \frac{-(50)(51)\mathbf{Y}_2\mathbf{I}_1}{(\mathbf{Y}_1 + \mathbf{Y}_2)(51\mathbf{Y}_2 + \mathbf{Y}_3) - 51\mathbf{Y}_2^2} = \mathbf{-2451.92\ E}_i$$

27. a.

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}$$
$$\frac{4 \times 10^3 \angle 0°}{4 \times 10^3 \angle 90°} \stackrel{?}{=} \frac{4 \times 10^3 \angle 0°}{4 \times 10^3 \angle -90°}$$
$$1 \angle -90° \neq 1 \angle 90° \text{ (not balanced)}$$

b. The solution to 26(b) resulted in

$$\mathbf{I}_3 = \mathbf{I}_{X_C} = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5))}{\mathbf{Z}_\Delta}$$

where $\mathbf{Z}_\Delta = (\mathbf{Z}_1 + \mathbf{Z}_3 + \mathbf{Z}_6)[(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{Z}_5^2]$
$- \mathbf{Z}_1[\mathbf{Z}_1(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{Z}_3\mathbf{Z}_5] - \mathbf{Z}_3[\mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)]$

and $\mathbf{Z}_1 = 5\text{ k}\Omega \angle 0°$, $\mathbf{Z}_2 = 8\text{ k}\Omega \angle 0°$, $\mathbf{Z}_3 = 2.5\text{ k}\Omega \angle 90°$
$\mathbf{Z}_4 = 4\text{ k}\Omega \angle 90°$, $\mathbf{Z}_5 = 5\text{ k}\Omega \angle -90°$, $\mathbf{Z}_6 = 1\text{ k}\Omega \angle 0°$

and $\mathbf{I}_{X_C} = \mathbf{1.76\text{ mA} \angle -71.54°}$

c. The solution to 26(c) resulted in

$$\mathbf{V}_3 = \mathbf{V}_{X_C} = \frac{\mathbf{I}[\mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)]}{\mathbf{Y}_\Delta}$$

where $\mathbf{Y}_\Delta = (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_6)[(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) - \mathbf{Y}_5^2]$
$- \mathbf{Y}_1\,[\mathbf{Y}_1(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) + \mathbf{Y}_2\mathbf{Y}_5]$
$- \mathbf{Y}_2[\mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)]$

with $\mathbf{Y}_1 = 0.2\text{ mS} \angle 0°$, $\mathbf{Y}_2 = 0.125\text{ mS} \angle 0°$, $\mathbf{Y}_3 = 0.4\text{ mS} \angle -90°$
$\mathbf{Y}_4 = 0.25\text{ mS} \angle -90°$, $\mathbf{Y}_5 = 0.2\text{ mS} \angle 90°$

Source conversion: $\mathbf{Y}_6 = 1\text{ mS} \angle 0°$, $\mathbf{I} = 10\text{ mA} \angle 0°$
and $\mathbf{V}_3 = \mathbf{7.03\text{ V} \angle -18.46°}$

29. $X_{C_1} = \frac{1}{\omega C} = \frac{1}{(1000\text{ rad/s})(3\ \mu\text{F})} = \frac{1}{3}\text{k}\Omega$

$\mathbf{Z}_1 = R_1 \| X_{C_1} \angle -90° = (2\text{ k}\Omega \angle 0°) \| \frac{1}{3}\text{ k}\Omega \angle -90° = 328.8\ \Omega \angle -80.54°$

$\mathbf{Z}_2 = R_2 \angle 0° = 0.5\text{ k}\Omega \angle 0°$, $\mathbf{Z}_3 = R_3 \angle 0° = 4\text{ k}\Omega \angle 0°$

$\mathbf{Z}_4 = R_x + jX_{L_x} = 1\text{ k}\Omega + j6\text{ k}\Omega$

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}$$

$$\frac{328.8\ \Omega \angle -80.54°}{4\text{ k}\Omega \angle 0°} \stackrel{?}{=} \frac{0.5\text{ k}\Omega \angle 0°}{6.083\ \Omega \angle 80.54°}$$

$82.2 \angle -80.54° \stackrel{\checkmark}{=} 82.2 \angle -80.54°$ **(balanced)**

31. For balance:

$$R_1(R_x + jX_{L_x}) = R_2(R_3 + jX_{L_3})$$

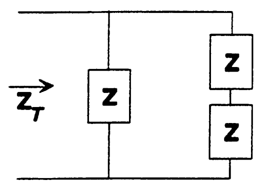
$$R_1R_x + jR_1X_{L_x} = R_2R_3 + jR_2X_{L_3}$$

$$\therefore\ R_1R_x = R_2R_3 \text{ and } R_x = \frac{R_2R_3}{R_1}$$

$$R_1X_{L_x} = R_2X_{L_3} \text{ and } R_1\omega L_x = R_2\omega L_3$$

$$\text{so that } L_x = \frac{R_2L_3}{R_1}$$

33. a.

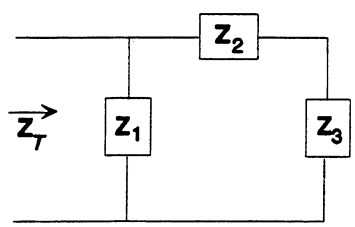


$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y = 3(3\ \Omega\ \angle 90°) = 9\ \Omega\ \angle 90°$$
$$\mathbf{Z} = 9\ \Omega\ \angle 90° \,\|\, (12\ \Omega - j16\ \Omega)$$
$$= 9\ \Omega\ \angle 90° \,\|\, 20\ \Omega\ \angle 53.13°$$
$$= 12.96\ \Omega\ \angle 67.13°$$

$$\mathbf{Z}_T = \mathbf{Z} \,\|\, 2\mathbf{Z} = \frac{2\mathbf{Z}^2}{\mathbf{Z} + 2\mathbf{Z}} = \frac{2}{3}\mathbf{Z} = \frac{2}{3}[12.96\ \Omega\ \angle 67.13°] = 8.64\ \Omega\ \angle 67.13°$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V}\ \angle 0°}{8.64\ \Omega\ \angle 67.13°} = \mathbf{11.57\ A\ \angle -67.13°}$$

b. $\mathbf{Z}_\Delta = 3\mathbf{Z}_Y = 3(5\ \Omega) = 15\ \Omega$



$$\mathbf{Z}_1 = 15\ \Omega\ \angle 0° \,\|\, 5\ \Omega\ \angle -90°$$
$$= 4.74\ \Omega\ \angle -71.57°$$
$$\mathbf{Z}_2 = 15\ \Omega\ \angle 0° \,\|\, 6\ \Omega\ \angle 90°$$
$$= 5.57\ \Omega\ \angle 68.2° = 2.07\ \Omega + j5.17\ \Omega$$
$$\mathbf{Z}_3 = \mathbf{Z}_1 = 4.74\ \Omega\ \angle -71.57°$$
$$= 1.5\ \Omega - j4.5\ \Omega$$

$$\mathbf{Z}_T = \mathbf{Z}_1 \,\|\, (\mathbf{Z}_2 + \mathbf{Z}_3) = (4.74\ \Omega\ \angle -71.57°) \,\|\, (2.07\ \Omega + j5.17\ \Omega + 1.5\ \Omega - j4.5\ \Omega)$$
$$= (4.74\ \Omega\ \angle -7.57°) \,\|\, (3.63\ \Omega\ \angle 10.63°)$$
$$= 2.71\ \Omega\ \angle -23.87°$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V}\ \angle 0°}{2.71\ \Omega\ \angle -23.87°} = \mathbf{36.9\ A\ \angle 23.87°}$$

CHAPTER 17 (Even)

2. a. $\mathbf{Z} = 5.6\ \Omega + j8.2\ \Omega = \mathbf{9.93\ \Omega\ \angle 55.67^\circ}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{20\ \text{V}\ \angle 20^\circ}{9.93\ \Omega\ \angle 55.67^\circ} = \mathbf{2.014\ A\ \angle -35.67^\circ}$$

b. $\mathbf{Z} = 2\ \Omega\ \angle 0^\circ \parallel 5\ \Omega\ \angle 90^\circ = \mathbf{1.86\ \Omega\ \angle 21.8^\circ}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{60\ \text{V}\ \angle 30^\circ}{1.86\ \Omega\ \angle 21.8^\circ} = 32.26\ \text{A}\ \angle 8.2^\circ$$

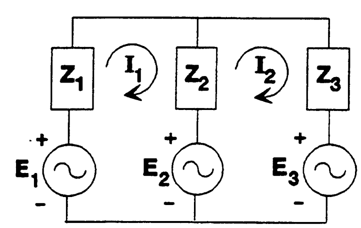
4. a. $$\mathbf{I} = \frac{\mu\mathbf{V}}{R} = \frac{16\ \text{V}}{4 \times 10^3} = \mathbf{4 \times 10^{-3}\ V}$$

$\mathbf{Z = 4\ k\Omega\ \angle 0^\circ}$

b. $\mathbf{V} = (h\mathbf{I})(R) = (50\ \mathbf{I})(50\ \text{k}\Omega) = \mathbf{2.5 \times 10^6\ I}$

$\mathbf{Z = 50\ k\Omega\ \angle 0^\circ}$

6. a.



$\mathbf{Z}_1 = 12\ \Omega + j12\ \Omega = 16.971\ \Omega\ \angle 45^\circ$
$\mathbf{Z}_2 = 3\ \Omega\ \angle 0^\circ$
$\mathbf{Z}_3 = -j1\ \Omega$
$\mathbf{E}_1 = 20\ \text{V}\ \angle 50^\circ$
$\mathbf{E}_2 = 60\ \text{V}\ \angle 70^\circ$
$\mathbf{E}_3 = 40\ \text{V}\ \angle 0^\circ$

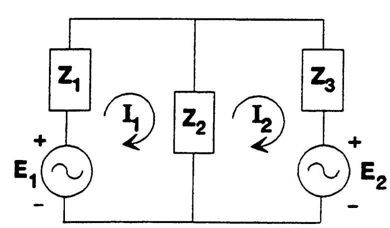
$$\mathbf{I}_1[\mathbf{Z}_1 + \mathbf{Z}_2] - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1 - \mathbf{E}_2$$
$$\mathbf{I}_2[\mathbf{Z}_2 + \mathbf{Z}_3] - \mathbf{Z}_2\mathbf{I}_1 = \mathbf{E}_2 - \mathbf{E}_3$$

$$(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1 - \mathbf{E}_2$$
$$-\mathbf{Z}_2\mathbf{I}_1 + (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_2 = \mathbf{E}_2 - \mathbf{E}_3$$

Using determinants:

$$\mathbf{I}_{R_1} = \mathbf{I}_1 = \frac{(\mathbf{E}_1 - \mathbf{E}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) + \mathbf{Z}_2(\mathbf{E}_2 - \mathbf{E}_3)}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} = \mathbf{2.552\ A\ \angle 132.72^\circ}$$

b.



Source conversion:

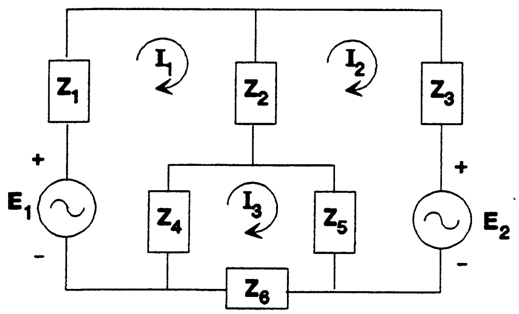
$\mathbf{E}_1 = \mathbf{IZ} = (6\ \text{A}\ \angle 0^\circ)(2\ \Omega\ \angle 0^\circ)$
$= 12\ \text{V}\ \angle 0^\circ$
$\mathbf{Z}_1 = 2\ \Omega + 20\ \Omega + j20\ \Omega = 22\ \Omega + j20\ \Omega$
$= 29.732\ \Omega\ \angle 42.274^\circ$
$\mathbf{Z}_2 = -j10\ \Omega = 10\ \Omega\ \angle -90^\circ$
$\mathbf{Z}_3 = 10\ \Omega\ \angle 0^\circ$

$$\begin{aligned} I_1[Z_1 + Z_2] - Z_2 I_2 &= E_1 \\ I_2[Z_2 + Z_3] - Z_2 I_1 &= -E_2 \end{aligned}$$

$$\begin{aligned} (Z_1 + Z_2)I_1 - Z_2 I_2 &= E_1 \\ -Z_2 I_1 + (Z_2 + Z_3)I_2 &= -E_2 \end{aligned}$$

$$I_{R_1} = I_1 = \frac{E_1(Z_2 + Z_3) - Z_2 E_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 0.495 \text{ A } \angle 72.255°$$

8. a.



$Z_1 = 5\ \Omega \angle 0°$, $Z_2 = 5\ \Omega \angle 90°$
$Z_3 = 4\ \Omega \angle 0°$, $Z_4 = 6\ \Omega \angle -90°$
$Z_5 = 4\ \Omega \angle 0°$, $Z_6 = 6\ \Omega + j8\ \Omega$
$E_1 = 20\text{ V} \angle 0°$, $E_2 = 40\text{ V} \angle 60°$

$$\begin{aligned} I_1(Z_1 + Z_2 + Z_4) - I_2 Z_2 - I_3 Z_4 &= E_1 \\ I_2(Z_2 + Z_3 + Z_5) - I_1 Z_2 - I_3 Z_5 &= -E_2 \\ I_3(Z_4 + Z_5 + Z_6) - I_1 Z_4 - I_2 Z_5 &= 0 \end{aligned}$$

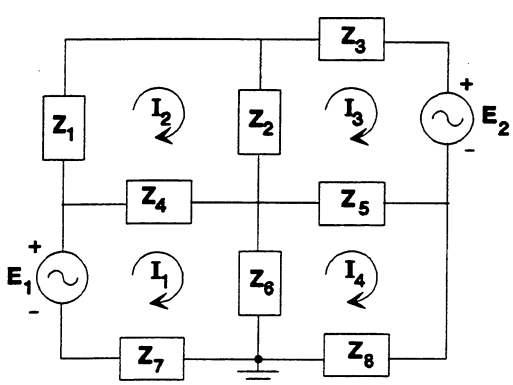
$$\begin{aligned} (Z_1 + Z_2 + Z_4)I_1 \quad - Z_2 I_2 \quad - Z_4 I_3 &= E_1 \\ -Z_2 I_1 + (Z_2 + Z_3 + Z_5)I_2 \quad - Z_5 I_3 &= -E_2 \\ -Z_4 I_1 \quad - Z_5 I_2 + (Z_4 + Z_5 + Z_6)I_3 &= 0 \end{aligned}$$

Using $Z' = Z_1 + Z_2 + Z_4$, $Z'' = Z_2 + Z_3 + Z_5$, $Z''' = Z_4 + Z_5 + Z_6$ and determinants:

$$I_{R_1} = I_1 = \frac{E_1(Z''Z''' - Z_5^2) - E_2(Z_2 Z''' + Z_4 Z_5)}{Z'(Z''Z''' - Z_5^2) - Z_2(Z_2 Z''' + Z_4 Z_5) - Z_4(Z_2 Z_5 + Z_4 Z'')}$$

$$= 3.04 \text{ A } \angle 169.12°$$

b.



$Z_1 = 10\ \Omega + j20\ \Omega$	$Z_2 = -j20\ \Omega$
$Z_3 = 80\ \Omega \angle 0°$	$Z_4 = 6\ \Omega \angle 0°$
$Z_5 = 15\ \Omega \angle 90°$	$Z_6 = 10\ \Omega \angle 0°$
$Z_7 = 5\ \Omega \angle 0°$	$Z_8 = 5\ \Omega - j20\ \Omega$
$E_1 = 25\text{ V} \angle 0°$	$E_2 = 75\text{ V} \angle 20°$

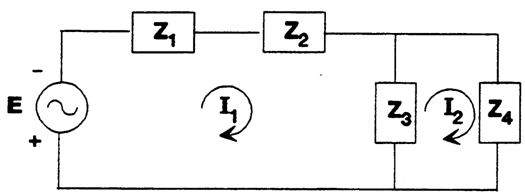
$$
\begin{aligned}
&\mathbf{I}_1(\mathbf{Z}_4 + \mathbf{Z}_6 + \mathbf{Z}_7) - \mathbf{I}_2\mathbf{Z}_4 - \mathbf{I}_4\mathbf{Z}_6 = \mathbf{E}_1\\
&\mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_4 - \mathbf{I}_3\mathbf{Z}_2 = 0\\
&\mathbf{I}_3(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_5) - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_4\mathbf{Z}_5 = -\mathbf{E}_2\\
&\mathbf{I}_4(\mathbf{Z}_5 + \mathbf{Z}_6 + \mathbf{Z}_8) - \mathbf{I}_1\mathbf{Z}_6 - \mathbf{I}_3\mathbf{Z}_5 = 0
\end{aligned}
$$

$$
\begin{array}{llll}
(\mathbf{Z}_4 + \mathbf{Z}_6 + \mathbf{Z}_7)\mathbf{I}_1 & - \mathbf{Z}_4\mathbf{I}_2 & + 0 & - \mathbf{Z}_6\mathbf{I}_4 = \mathbf{E}_1\\
-\mathbf{Z}_4\mathbf{I}_1 & + (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4)\mathbf{I}_2 & - \mathbf{Z}_2\mathbf{I}_3 & + 0 = 0\\
0 & - \mathbf{Z}_2\mathbf{I}_2 & + (\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_5)\mathbf{I}_3 & - \mathbf{Z}_5\mathbf{I}_4 = -\mathbf{E}_2\\
-\mathbf{Z}_6\mathbf{I}_1 & + 0 & - \mathbf{Z}_5\mathbf{I}_3 & + (\mathbf{Z}_5 + \mathbf{Z}_6 + \mathbf{Z}_7)\mathbf{I}_4 = 0
\end{array}
$$

Applying determinants:

$$\mathbf{I}_{R_1} = \mathbf{I}_{80\Omega} = 0.681\text{ A } \angle -162.9^\circ$$

10.



Source Conversion:

$$
\begin{aligned}
\mathbf{E} &= (I \angle\theta)(R_p \angle 0^\circ)\\
&= (50\ \mathbf{I})(40\text{ k}\Omega \angle 0^\circ)\\
&= 2 \times 10^6\ \mathbf{I} \angle 0^\circ
\end{aligned}
$$

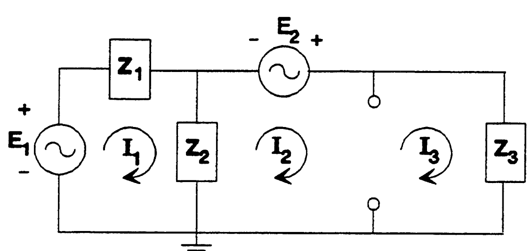
$\mathbf{Z}_1 = R_s = R_p = 40\text{ k}\Omega \angle 0^\circ$
$\mathbf{Z}_2 = -j0.2\text{ k}\Omega$
$\mathbf{Z}_3 = 8\text{ k}\Omega \angle 0^\circ$
$\mathbf{Z}_4 = 4\text{ k}\Omega \angle 90^\circ$

$$
\begin{aligned}
&\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_3\mathbf{I}_2 = -\mathbf{E}\\
&\mathbf{I}_2(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{Z}_3\mathbf{I}_1 = 0
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_1 - \mathbf{Z}_3\mathbf{I}_2 = -\mathbf{E}\\
&-\mathbf{Z}_3\mathbf{I}_1 + (\mathbf{Z}_3 + \mathbf{Z}_4)\mathbf{I}_2 = 0
\end{aligned}
$$

$$\mathbf{I}_L = \mathbf{I}_2 = \frac{-\mathbf{Z}_3\mathbf{E}}{(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{Z}_3^2} = 42.91\ \mathbf{I} \angle 149.31^\circ$$

12.



$\mathbf{E}_1 = 5\text{ V} \angle 0^\circ$
$\mathbf{E}_2 = 20\text{ V} \angle 0^\circ$
$\mathbf{Z}_1 = 2.2\text{ k}\Omega \angle 0^\circ$
$\mathbf{Z}_2 = 5\text{ k}\Omega \angle 90^\circ$
$\mathbf{Z}_3 = 10\text{ k}\Omega \angle 0^\circ$
$\mathbf{I} = 4\text{ mA} \angle 0^\circ$

$$
\begin{aligned}
&\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0\\
&-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{E}_2 - \mathbf{I}_3\mathbf{Z}_3 = 0
\end{aligned}
$$

$$\mathbf{I}_3 - \mathbf{I}_2 = \mathbf{I}$$

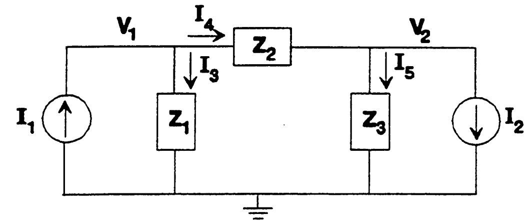
Substituting, we obtain:

$$
\begin{aligned}
&\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1\\
&\mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) = \mathbf{I}\mathbf{Z}_3 - \mathbf{E}_2
\end{aligned}
$$

Determinants:

$$\mathbf{I}_1 = 1.39\text{ mA} \angle -126.48^\circ,\ \mathbf{I}_2 = 1.341\text{ mA} \angle -10.56^\circ,\ \mathbf{I}_3 = 2.693\text{ mA} \angle -174.8^\circ$$
$$\mathbf{I}_{10\text{k}\Omega} = \mathbf{I}_3 = 2.693\text{ mA} \angle -174.8^\circ$$

14. a.



$Z_1 = 4\ \Omega\ \angle 0°$
$Z_2 = 5\ \Omega\ \angle 90°$
$Z_3 = 2\ \Omega\ \angle -90°$
$I_1 = 3\ A\ \angle 0°$
$I_2 = 5\ A\ \angle 30°$

$$I_1 = I_3 + I_4$$

$$I_1 = \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} \Rightarrow V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2}\right] - V_2\left[\frac{1}{Z_2}\right] = I_1$$

or $V_1[Y_1 + Y_2] - V_2[Y_2] = I_1$

$$I_4 = I_5 + I_2$$

$$\frac{V_1 - V_2}{Z_2} = \frac{V_2}{Z_3} + I_2 \Rightarrow V_2\left[\frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_1\left[\frac{1}{Z_2}\right] = -I_2$$

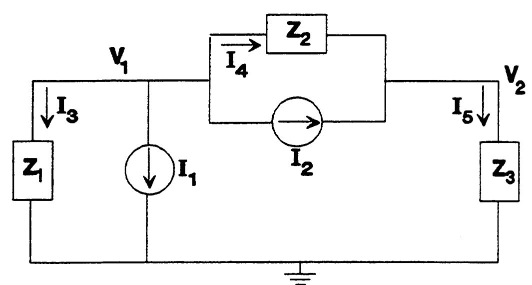
or $V_2[Y_2 + Y_3] - V_1[Y_2] = -I_2$

$$\begin{aligned} [Y_1 + Y_2]V_1 \quad\quad\quad - Y_2V_2 &= I_1 \\ -Y_2V_1 + [Y_2 + Y_3]V_2 &= -I_2 \end{aligned}$$

$$V_1 = \frac{[Y_2 + Y_3]I_1 - Y_2I_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} = \mathbf{14.68\ V\ \angle 68.89°}$$

$$V_2 = \frac{-[Y_2 + Y_2]I_2 + Y_2I_1}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} = \mathbf{12.97\ V\ \angle 155.88°}$$

b.



$Z_1 = 3\ \Omega + j4\ \Omega = 5\ \angle 53.13°$
$Z_2 = 2\ \Omega\ \angle 0°$
$Z_3 = 6\ \Omega\ \angle 0° \| 8\ \Omega\ \angle -90°$
$\quad = 4.8\ \Omega\ \angle -36.87°$
$I_1 = 0.6\ A\ \angle 20°$
$I_2 = 4\ A\ \angle 80°$

$$0 = I_1 + I_3 + I_4 + I_2$$

$$0 = I_1 + \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} + I_2$$

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2}\right] - V_2\left[\frac{1}{Z_2}\right] = -I_1 - I_2$$

or $\quad V_1[Y_1 + Y_2] - V_2[Y_2] = -I_1 - I_2$

$$I_2 + I_4 = I_5$$

$$I_2 + \frac{V_1 - V_2}{Z_2} = \frac{V_2}{Z_3}$$

$$V_2\left[\frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_1\left[\frac{1}{Z_2}\right] = +I_2$$

or $\quad V_2[Y_2 + Y_3] - V_1[Y_2] = I_2$

and $\quad [Y_1 + Y_2]V_1 - Y_2V_2 = -I_1 - I_2$

$$-Y_2V_1 + [Y_2 + Y_3]V_2 = I_2$$

Applying determinants:

$$V_1 = \frac{-[Y_2 + Y_3][I_1 + I_2] + Y_2I_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} = \mathbf{5.12\ V\ \angle -79.36^\circ}$$

$$V_2 = \frac{Y_1I_2 - I_1Y_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} = \mathbf{2.71\ V\ \angle 39.96^\circ}$$

16. $$I = \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2}$$

$$0 = \frac{V_2 - V_1}{Z_2} + \frac{V_2}{Z_3} + \frac{V_2 - E}{Z_4}$$

$Z_1 = 2\ \Omega\ \angle 0^\circ$
$Z_2 = 20\ \Omega + j\,20\ \Omega$
$Z_3 = 10\Omega\ \angle -90^\circ$
$Z_4 = 10\ \Omega\ \angle 0^\circ$
$I = 6\ A\ \angle 0^\circ$
$E = 30\ V\ \angle 0^\circ$

Rearranging:

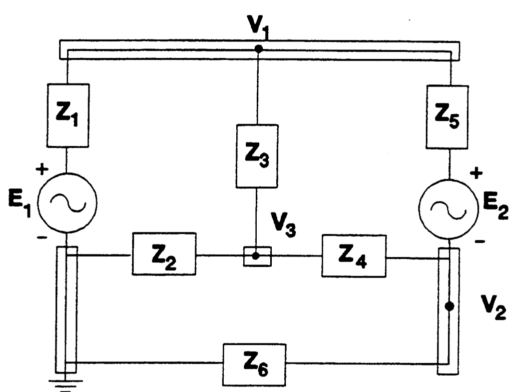
$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2}\right] - \frac{1}{Z_2}V_2 = I$$

$$\frac{-V_1}{Z_2} + V_2\left[\frac{1}{Z_2} + \frac{1}{Z_2} + \frac{1}{Z_4}\right] = \frac{E}{Z_4}$$

Determinants and substituting:

$$V_1 = \mathbf{11.74\ V\ \angle -4.611^\circ},\ V_2 = \mathbf{22.53\ V\ \angle -36.48^\circ}$$

18.



$Z_1 = 5\ \Omega\ \angle 0^\circ$
$Z_2 = 6\ \Omega\ \angle -90^\circ$
$Z_3 = 5\ \Omega\ \angle 90^\circ$
$Z_4 = 4\ \Omega\ \angle 0^\circ$
$Z_5 = 4\ \Omega\ \angle 0^\circ$
$Z_6 = 6\ \Omega + j8\ \Omega$
$E_1 = 20\ V\ \angle 0^\circ$
$E_2 = 40\ V\ \angle 60^\circ$

node V_1: $\dfrac{V_1 - E_1}{Z_1} + \dfrac{V_1 - V_3}{Z_3} + \dfrac{V_1 - E_2 - V_2}{Z_5} = 0$

node V_2: $\dfrac{V_2 + E_2 - V_1}{Z_5} + \dfrac{V_2 - V_3}{Z_4} + \dfrac{V_2}{Z_6} = 0$

node V_3: $\dfrac{V_3}{Z_2} + \dfrac{V_3 - V_1}{Z_3} + \dfrac{V_3 - V_2}{Z_4} = 0$

Rearranging:

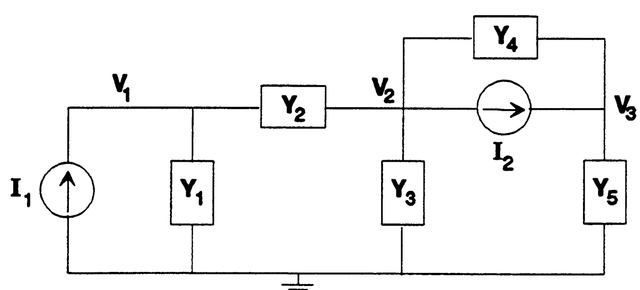
$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_3} + \frac{1}{Z_5}\right] - \frac{V_2}{Z_5} - \frac{V_3}{Z_3} = \frac{E_1}{Z_1} + \frac{E_2}{Z_5}$$

$$V_2\left[\frac{1}{Z_5} + \frac{1}{Z_4} + \frac{1}{Z_6}\right] - \frac{V_1}{Z_5} - \frac{V_3}{Z_4} = -\frac{E_2}{Z_5}$$

$$V_3\left[\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4}\right] - \frac{V_1}{Z_3} - \frac{V_2}{Z_4} = 0$$

Determinants: $V_1 = 5.839 \text{ V} \angle 29.4°$, $V_2 = 28.06 \text{ V} \angle -89.15°$, $V_3 = 31.96 \text{ V} \angle -77.6$

20. a.



$Y_1 = \dfrac{1}{4\ \Omega \angle 0°} = 0.25 \text{ S} \angle 0°$

$Y_2 = \dfrac{1}{1\ \Omega \angle 90°} = 1 \text{ S} \angle -90°$

$Y_3 = \dfrac{1}{5\ \Omega \angle 0°} = 0.2 \text{ S} \angle 0°$

$Y_4 = \dfrac{1}{4\ \Omega \angle -90°} = 0.25 \text{ S} \angle 90°$

$Y_5 = \dfrac{1}{8\ \Omega \angle 90°} = 0.125 \text{ S} \angle -90°$

$I_1 = 2 \text{ A} \angle 30°$

$I_2 = 3 \text{ A} \angle 150°$

$V_1[Y_1 + Y_2] - Y_2V_2 = I_1$

$V_2[Y_2 + Y_3 + Y_4] - Y_2V_1 - Y_4V_3 = -I_2$

$V_3[Y_4 + Y_5] - Y_4V_2 = I_2$

$$\begin{aligned}
&[Y_1 + Y_2]V_1 && - Y_2V_2 && + 0 = I_1 \\
&-Y_2V_1 && + [Y_2 + Y_3 + Y_4]V_2 && - Y_4V_3 = -I_2 \\
&0 && - Y_4V_2 && + [Y_4 + Y_5]V_3 = I_2
\end{aligned}$$

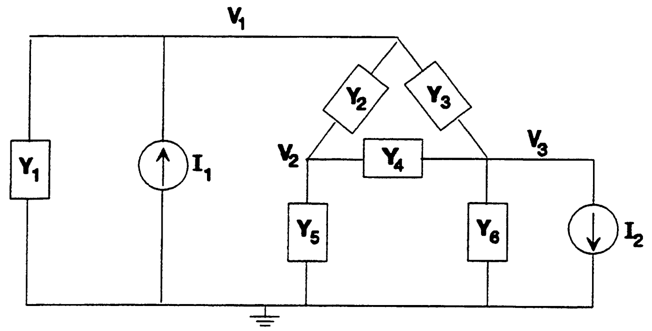
$$V_1 = \frac{I_1\left[(Y_2 + Y_3 + Y_4)(Y_4 + Y_5) - Y_4^2\right] - I_2[Y_2Y_5]}{[Y_1 + Y_2]\left[(Y_2 + Y_3 + Y_4)(Y_4 + Y_5) - Y_4^2\right] - Y_2^2(Y_4 + Y_5) = Y_\Delta}$$

$= 5.74 \text{ V} \angle 122.76°$

$$\mathbf{V}_2 = \frac{\mathbf{I}_1\mathbf{Y}_2(\mathbf{Y}_4 + \mathbf{Y}_5) - \mathbf{I}_2\mathbf{Y}_5(\mathbf{Y}_1 + \mathbf{Y}_2)}{\mathbf{Y}_\Delta} = \mathbf{4.04\ V\ \angle 145.03^\circ}$$

$$\mathbf{V}_3 = \frac{\mathbf{I}_2\left[(\mathbf{Y}_1 + \mathbf{Y}_2)(\mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{Y}_2^2\right] - \mathbf{Y}_2\mathbf{Y}_4\mathbf{I}_1}{\mathbf{Y}_\Delta} = \mathbf{25.94\ V\ \angle 78.07^\circ}$$

b.



$$\mathbf{Y}_1 = \frac{1}{4\ \Omega\ \angle 0^\circ} = 0.25\ \text{S}\ \angle 0^\circ$$
$$\mathbf{Y}_2 = \frac{1}{6\ \Omega\ \angle 0^\circ} = 0.167\ \text{S}\ \angle 0^\circ$$
$$\mathbf{Y}_3 = \frac{1}{8\ \Omega\ \angle 0^\circ} = 0.125\ \text{S}\ \angle 0^\circ$$
$$\mathbf{Y}_4 = \frac{1}{2\ \Omega\ \angle -90^\circ} = 0.5\ \text{S}\ \angle 90^\circ$$
$$\mathbf{Y}_5 = \frac{1}{5\ \Omega\ \angle 90^\circ} = 0.2\ \text{S}\ \angle -90^\circ$$
$$\mathbf{Y}_6 = \frac{1}{4\ \Omega\ \angle 90^\circ} = 0.25\ \text{S}\ \angle -90^\circ$$
$$\mathbf{I}_1 = 4\ \text{A}\ \angle 0^\circ$$
$$\mathbf{I}_2 = 6\ \text{A}\ \angle 90^\circ$$

$$\begin{aligned}
&\mathbf{V}_1[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3] - \mathbf{Y}_2\mathbf{V}_2 - \mathbf{Y}_3\mathbf{V}_3 = \mathbf{I}_1\\
&\mathbf{V}_2[\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5] - \mathbf{Y}_2\mathbf{V}_1 - \mathbf{Y}_4\mathbf{V}_3 = 0\\
&\mathbf{V}_3[\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_6] - \mathbf{Y}_3\mathbf{V}_1 - \mathbf{Y}_4\mathbf{V}_2 = -\mathbf{I}_2
\end{aligned}$$

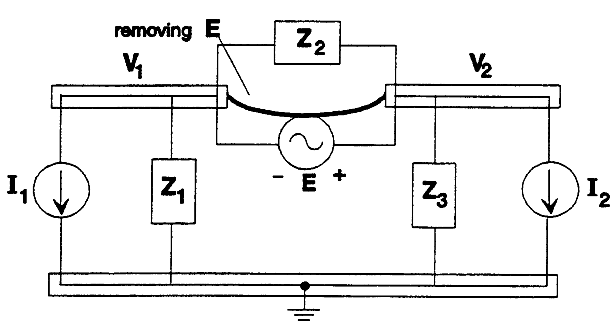
$$\begin{aligned}
[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3]\mathbf{V}_1 \quad & - \mathbf{Y}_2\mathbf{V}_2 & - \mathbf{Y}_3\mathbf{V}_3 &= \mathbf{I}_1\\
-\mathbf{Y}_2\mathbf{V}_1 \quad & + [\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5]\mathbf{V}_2 & - \mathbf{Y}_4\mathbf{V}_3 &= 0\\
-\mathbf{Y}_3\mathbf{V}_1 \quad & - \mathbf{Y}_4\mathbf{V}_2 & + [\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_6]\mathbf{V}_3 &= -\mathbf{I}_2
\end{aligned}$$

$$\mathbf{V}_1 = \frac{\mathbf{I}_1\left[(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)(\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_6) - \mathbf{Y}_4^2\right] - \mathbf{I}_2[\mathbf{Y}_2\mathbf{Y}_4 + \mathbf{Y}_3(\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_5)]}{\mathbf{Y}_\Delta = (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3)\left[(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)(\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_6) - \mathbf{Y}_4^2\right] - \mathbf{Y}_2[\mathbf{Y}_2(\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_6) + \mathbf{Y}_3\mathbf{Y}_4] - \mathbf{Y}_3[\mathbf{Y}_2\mathbf{Y}_4 + \mathbf{Y}_3(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)]}$$
$$= \mathbf{15.13\ V\ \angle 1.29^\circ}$$

$$\mathbf{V}_2 = \frac{\mathbf{I}_1[(\mathbf{Y}_2)(\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_6) + \mathbf{Y}_3\mathbf{Y}_4] + \mathbf{I}_2[\mathbf{Y}_4(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{Y}_2\mathbf{Y}_3]}{\mathbf{Y}_\Delta} = \mathbf{17.24\ V\ \angle 3.73^\circ}$$

$$\mathbf{V}_3 = \frac{\mathbf{I}_1[(\mathbf{Y}_3)(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) + \mathbf{Y}_2\mathbf{Y}_4] + \mathbf{I}_2\left[\mathbf{Y}_2^2 - (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3)(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)\right]}{\mathbf{Y}_\Delta}$$
$$= \mathbf{10.59\ V\ \angle -0.11^\circ}$$

22.



$$\mathbf{Z}_1 = 1\ \text{k}\Omega\ \angle 0^\circ$$
$$\mathbf{Z}_2 = 2\ \text{k}\Omega\ \angle 90^\circ$$
$$\mathbf{Z}_3 = 3\ \text{k}\Omega\ \angle -90^\circ$$
$$\mathbf{I}_1 = 12\ \text{mA}\ \angle 0^\circ$$
$$\mathbf{I}_2 = 4\ \text{mA}\ \angle 0^\circ$$
$$\mathbf{E} = 10\ \text{V}\ \angle 0^\circ$$

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

$$0 = \mathbf{I}_1 + \frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + \mathbf{I}_2$$

and $\dfrac{\mathbf{V}_1}{\mathbf{Z}_1} + \dfrac{\mathbf{V}_2}{\mathbf{Z}_3} = -\mathbf{I}_1 - \mathbf{I}_2$

with $\mathbf{V}_2 - \mathbf{V}_1 = \mathbf{E}$

Substituting and rearranging:

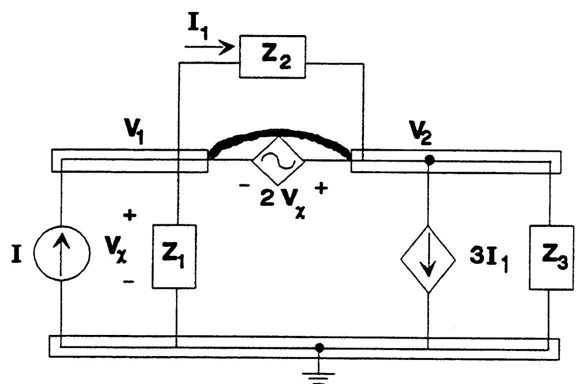
$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_3}\right] = -\mathbf{I}_1 - \mathbf{I}_2 - \frac{\mathbf{E}}{\mathbf{Z}_3}$$

and solving for $\mathbf{V}_1$:

$\mathbf{V}_1 = \mathbf{15.4\ V} \angle \mathbf{178.2°}$

with $\mathbf{V}_2 = \mathbf{5.414\ V} \angle \mathbf{174.87°}$

24.



$\mathbf{Z}_1 = 2\ \text{k}\Omega \angle 0°$
$\mathbf{Z}_2 = 1\ \text{k}\Omega \angle 0°$
$\mathbf{Z}_3 = 1\ \text{k}\Omega \angle 0°$
$\mathbf{I} = 5\ \text{mA} \angle 0°$

$\mathbf{V}_1$: $\mathbf{I} = \dfrac{\mathbf{V}_1}{\mathbf{Z}_1} + 3\mathbf{I}_1 + \dfrac{\mathbf{V}_2}{\mathbf{Z}_3}$

with $\mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2}$

and $\mathbf{V}_2 - \mathbf{V}_1 = 2\mathbf{V}_x = 2\mathbf{V}_1$ or $\mathbf{V}_2 = 3\mathbf{V}_1$

Substituting with result in:

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{3}{\mathbf{Z}_2}\right] + 3\ \mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3} - \frac{3}{\mathbf{Z}_2}\right] = \mathbf{I}$$

or $\mathbf{V}_1\left[\dfrac{1}{\mathbf{Z}_1} - \dfrac{6}{\mathbf{Z}_2} + \dfrac{3}{\mathbf{Z}_3}\right] = \mathbf{I}$

and $\mathbf{V}_1 = -2\ \text{V} \angle 0°$

with $\mathbf{V}_2 = -6\ \text{V} \angle 0°$

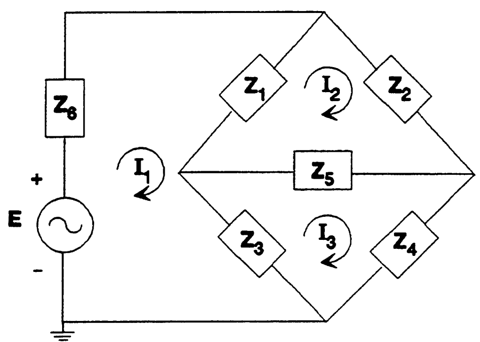
26. a. yes

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}$$

$$\frac{5 \times 10^3 \angle 0^\circ}{2.5 \times 10^3 \angle 90^\circ} = \frac{8 \times 10^3 \angle 0^\circ}{4 \times 10^3 \angle 90^\circ}$$

$$2 \angle -90^\circ = 2 \angle -90^\circ \text{ (balanced)}\checkmark$$

b. $\mathbf{Z}_1 = 5\text{ k}\Omega \angle 0^\circ, \mathbf{Z}_2 = 8\text{ k}\Omega \angle 0^\circ$
$\mathbf{Z}_3 = 2.5\text{ k}\Omega \angle 90^\circ, \mathbf{Z}_4 = 4\text{ k}\Omega \angle 90^\circ$
$\mathbf{Z}_5 = 5\text{ k}\Omega \angle -90^\circ, \mathbf{Z}_6 = 1\text{ k}\Omega \angle 0^\circ$



$$\mathbf{I}_1[\mathbf{Z}_1 + \mathbf{Z}_3 + \mathbf{Z}_6] - \mathbf{Z}_1\mathbf{I}_2 - \mathbf{Z}_3\mathbf{I}_3 = \mathbf{E}$$
$$\mathbf{I}_2[\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5] - \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_5\mathbf{I}_3 = 0$$
$$\mathbf{I}_3[\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5] - \mathbf{Z}_3\mathbf{I}_1 - \mathbf{Z}_5\mathbf{I}_2 = 0$$

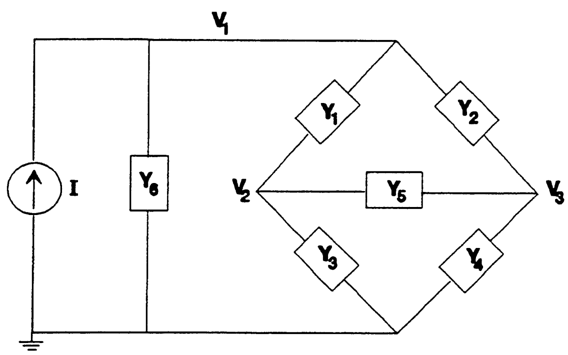
$$\begin{aligned}
[\mathbf{Z}_1 + \mathbf{Z}_3 + \mathbf{Z}_6]\mathbf{I}_1 \quad & - \mathbf{Z}_1\mathbf{I}_2 & - \mathbf{Z}_3\mathbf{I}_3 &= \mathbf{E}\\
-\mathbf{Z}_1\mathbf{I}_1 + [\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5]\mathbf{I}_2 & & - \mathbf{Z}_5\mathbf{I}_3 &= 0\\
-\mathbf{Z}_3\mathbf{I}_1 \quad & - \mathbf{Z}_5\mathbf{I}_2 + [\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5]\mathbf{I}_3 & &= 0
\end{aligned}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}[\mathbf{Z}_1(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) + \mathbf{Z}_3\mathbf{Z}_5]}{\mathbf{Z}_\Delta = (\mathbf{Z}_1 + \mathbf{Z}_3 + \mathbf{Z}_6)[(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{Z}_5^2] - \mathbf{Z}_1[\mathbf{Z}_1(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{Z}_3\mathbf{Z}_5] - \mathbf{Z}_3[\mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)]}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}[\mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)]}{\mathbf{Z}_\Delta}$$

$$\mathbf{I}_{Z_5} = \mathbf{I}_2 - \mathbf{I}_3 = \frac{\mathbf{E}[\mathbf{Z}_1\mathbf{Z}_4 - \mathbf{Z}_3\mathbf{Z}_2]}{\mathbf{Z}_\Delta} = \frac{\mathbf{E}[20 \times 10^6 \angle 90^\circ - 20 \times 10^6 \angle 90^\circ]}{\mathbf{Z}_\Delta} = \mathbf{0\ A}$$

c.



$$\mathbf{I} = \frac{\mathbf{E}_s}{\mathbf{R}_s} = \frac{10\text{ V}\angle 0^\circ}{1\text{ k}\Omega\angle 0^\circ} = 10\text{ mA}\angle 0^\circ$$

$$\mathbf{Y}_1 = \frac{1}{5\text{ k}\Omega\angle 0^\circ} = 0.2\text{ mS}\angle 0^\circ$$

$$\mathbf{Y}_2 = \frac{1}{8\text{ k}\Omega\angle 0^\circ} = 0.125\text{ mS}\angle 0^\circ$$

$$\mathbf{Y}_3 = \frac{1}{2.5\text{ k}\Omega\angle 90^\circ} = 0.4\text{ mS}\angle -90^\circ$$

$$\mathbf{Y}_4 = \frac{1}{4\text{ k}\Omega\angle 90^\circ} = 0.25\text{ mS}\angle -90^\circ$$

$$\mathbf{Y}_5 = \frac{1}{5\text{ k}\Omega\angle -90^\circ} = 0.2\text{ mS}\angle 90^\circ$$

$$\mathbf{Y}_6 = \frac{1}{1\text{ k}\Omega\angle 0^\circ} = 1\text{ mS}\angle 0^\circ$$

$$\begin{aligned}
\mathbf{V}_1[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_6] - \mathbf{Y}_1\mathbf{V}_2 - \mathbf{Y}_2\mathbf{V}_3 &= \mathbf{I}\\
\mathbf{V}_2[\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5] - \mathbf{Y}_1\mathbf{V}_1 - \mathbf{Y}_5\mathbf{V}_3 &= 0\\
\mathbf{V}_3[\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5] - \mathbf{Y}_2\mathbf{V}_1 - \mathbf{Y}_5\mathbf{V}_2 &= 0
\end{aligned}$$

$$\begin{aligned}
[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_6]\mathbf{V}_1 \quad - \mathbf{Y}_1\mathbf{V}_2 \quad - \mathbf{Y}_2\mathbf{V}_3 &= \mathbf{I}\\
-\mathbf{Y}_1\mathbf{V}_1 + [\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5]\mathbf{V}_2 \quad - \mathbf{Y}_5\mathbf{V}_3 &= 0\\
-\mathbf{Y}_2\mathbf{V}_1 \quad - \mathbf{Y}_5\mathbf{V}_2 + [\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5]\mathbf{V}_3 &= 0
\end{aligned}$$

$$\mathbf{V}_2 = \frac{\mathbf{I}[\mathbf{Y}_1(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) + \mathbf{Y}_2\mathbf{Y}_5]}{\mathbf{Y}_\Delta = (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_6)[(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) - \mathbf{Y}_5^2] - \mathbf{Y}_1[\mathbf{Y}_1(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) + \mathbf{Y}_2\mathbf{Y}_5] - \mathbf{Y}_2[\mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)]}$$

$$\mathbf{V}_3 = \frac{\mathbf{I}[(\mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)]}{\mathbf{Y}_\Delta}$$

$$\mathbf{V}_{Z_5} = \mathbf{V}_2 - \mathbf{V}_3 = \frac{\mathbf{I}[\mathbf{Y}_1\mathbf{Y}_4 - \mathbf{Y}_2\mathbf{Y}_3]}{\mathbf{Y}_\Delta} = \frac{\mathbf{I}[0.05 \times 10^{-3}\angle -90^\circ - 0.05 \times 10^{-3}\angle -90^\circ]}{\mathbf{Y}_\Delta}$$
$$= \mathbf{0\ V}$$

28. $\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2$

$$(R_1 - jX_C)(R_x + jX_{L_x}) = R_3R_2 \qquad X_C = \frac{1}{\omega C} = \frac{1}{(10^3\text{ rad/s})(1\ \mu\text{F})} = 1\text{ k}\Omega$$

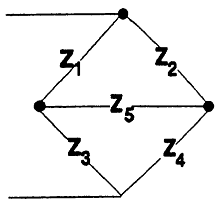
$$(1\text{ k}\Omega - j1\text{ k}\Omega)(R_x + jX_{L_x}) = (0.1\text{ k}\Omega)(0.1\text{ k}\Omega) = 10\text{ k}\Omega$$

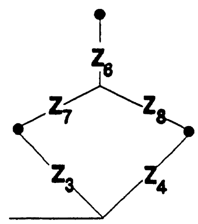
and $$R_x + jX_{L_x} = \frac{10 \times 10^3\ \Omega}{1 \times 10^3 - j1 \times 10^3} = \frac{10 \times 10^3}{1.414 \times 10^3\angle -45^\circ} = 5\ \Omega + j5\ \Omega$$

$$\therefore R_x = 5\ \Omega,\ L_x = \frac{X_{L_x}}{\omega} = \frac{5\ \Omega}{10^3\text{ rad/s}} = 5\text{ mH}$$

30. Apply Eq. **17.6.**

32. a.

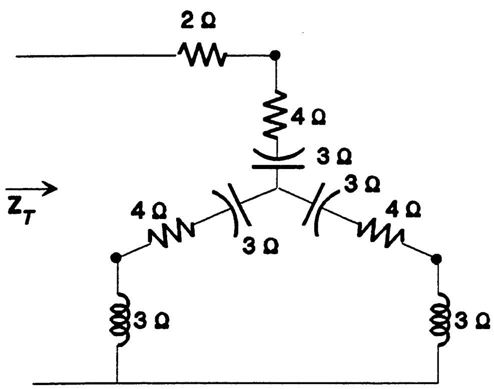




$\mathbf{Z}_1 = 8\ \Omega\ \angle -90^\circ = -j8\ \Omega$
$\mathbf{Z}_2 = 4\ \Omega\ \angle 90^\circ = +j4\ \Omega$
$\mathbf{Z}_3 = 8\ \Omega\ \angle 90^\circ = +j8\ \Omega$
$\mathbf{Z}_4 = 6\ \Omega\ \angle -90^\circ = -j6\ \Omega$
$\mathbf{Z}_5 = 5\ \Omega\ \angle 0^\circ$

$$\mathbf{Z}_6 = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5} = 5\ \Omega\ \angle 38.66^\circ$$
$$\mathbf{Z}_7 = \frac{\mathbf{Z}_1\mathbf{Z}_5}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5} = 6.25\ \Omega\ \angle -51.34^\circ$$
$$\mathbf{Z}_8 = \frac{\mathbf{Z}_2\mathbf{Z}_5}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5} = 3.125\ \Omega\ \angle 128.66^\circ$$
$$\mathbf{Z}' = \mathbf{Z}_7 + \mathbf{Z}_3 = 3.9\ \Omega + j3.12\ \Omega = 4.99\ \Omega\ \angle 38.66^\circ$$
$$\mathbf{Z}'' = \mathbf{Z}_8 + \mathbf{Z}_4 = -1.95\ \Omega - j3.56\ \Omega = 4.06\ \Omega\ \angle -118.71^\circ$$
$$\mathbf{Z}' \| \mathbf{Z}'' = 10.13\ \Omega\ \angle -67.33^\circ = 3.90\ \Omega - j9.35\ \Omega$$
$$\mathbf{Z}_T = \mathbf{Z}_6 + \mathbf{Z}' \| \mathbf{Z}'' = 7.80\ \Omega - j6.23\ \Omega = 9.98\ \Omega\ \angle -38.61^\circ$$
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\ \text{V}\ \angle 0^\circ}{9.98\ \Omega\ \angle -38.61^\circ} = \mathbf{12.02\ A\ \angle 38.61^\circ}$$

b. $$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = \frac{12\ \Omega - j9\ \Omega}{3} = 4\ \Omega - j3\ \Omega$$



$$\mathbf{Z}_T = 2\ \Omega + 4\ \Omega + j3\ \Omega + [4\ \Omega - j\cancel{3}\ \Omega + j\cancel{3}\ \Omega] \| [4\ \Omega - j\cancel{3}\ \Omega + j\cancel{3}\ \Omega]$$
$$= 6\ \Omega - j3\ \Omega + 2\ \Omega$$
$$= 8\ \Omega - j3\ \Omega = 8.544\ \Omega\ \angle -20.56^\circ$$
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{60\ \text{V}\ \angle 0^\circ}{8.544\ \Omega\ \angle -20.56^\circ} = \mathbf{7.02\ A\ \angle 20.56^\circ}$$